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INFLUENCE OF AXIAL DEFORMATION ON MODAL STRAIN ENERGY

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ABSTRACT

Axial loads of load bearing elements impact on the vibration characteristics. Several methods have been developed to quantify axial loads and hence axial deformations of individual structural members using their natural frequencies. Nevertheless, these methods cannot be applied to individual members in structural framing systems as the natural frequency is a global parameter for the entire framing system. This paper proposes an innovative method which uses modal strain energy phenomenon to quantify axial deformations of load bearing elements of structural framing systems. The procedure is illustrated through examples and results confirm that the proposed method has an ability to quantify the axial deformations of individual elements of structural framing systems

INTRODUCTION

Investigating the behaviour of axially loaded elements under the vibration is important as axial loads in members influence significantly on the vibration characteristics of structural framing system. Several researchers have investigated the influence of axial loads on vibration characteristics of individual loaded elements with different boundary conditions. Results concluded that a tensile axial load increases the natural frequencies, whereas the compressive load decreases the natural frequencies. These researchers established relationships between the frequency and the axial load. However, as the natural frequency is a property of entire structure, these methods are limited to individual elements. This is the main drawback of previous developments. Moreover, outcome of previous researches highlighted that the compressive load influences significantly on the first vibration mode shape of the beam and the influence on the second vibration mode is less significant (Banerjee, 2000; Della & Shu, 2009).

Based on the above, it is evident that the axial force influences the modal parameters especially the first few natural frequencies and the corresponding modal vectors. Modal vectors can be used to study the individual behavior of axially loaded elements of structural framing system since behaviors of these vectors are different for each element. According to the best of authors' knowledge, there is no method presently available to calculate axial deformation as a resulting of axial force of load bearing element in a structural framing system using modal vectors. This paper proposes an innovative procedure to determine axial deformation using Modal Strain Energy (MSE) phenomenon consisting of modal vectors.

METHODOLOGY

Finite element package, ANSYS (ANSYS Manual, v 11) is modified to capture stiffness change occurring due to the applied axial loads and incorporate into the modal analysis. Using the modified program, a FE model was developed for an element used to study the effect of axial force from previous publication (Banerjee, 2000). Analysis results were then compared with the previous publication and found a satisfactory agreement confirming the accuracy of the modified program.

Modal Strain Energy (MSE) method developed for beam like structures has been modified and used for the proposed method presenting in this paper. MSE is a well established method. This method uses to capture the damages/defects in civil engineering structures (Stubbs et al, 1992).

The energy associated with a particular mode shape, $\Psi_i(x)$ at distance x. can be written as

$$U_i^U = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 \Psi_i^U}{\partial x^2}\right)^2 dx \tag{1}$$

Where EI is the flexural rigidity of the beam and superscript U denotes the unloaded case.

If the beam is subdivided into N_d divisions, the energy associated with each sub division, j due to the i^{th} mode can be written as

$$U_{ij}^u = \frac{1}{2}\int_{a_j}^{a_{j+1}} EI_j\left(\frac{\partial^2 \Psi_i^U}{\partial x^2}\right)^2 dx \tag{2}$$

The fractional energy is thereby

$$F_{ij}^U = \frac{U_{ij}^U}{U_j^U} \text{ and } \sum_{j=1}^{N_d} F_{ij}^U = 1 \tag{3}$$

MSE of the beam element after being subjected to the axial load (loaded case) can be written the same as the unload case as follows.

$$U^L{}_i = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 \Psi^L{}_i}{\partial x^2}\right)^2 dx \text{ and } U^L{}_{ij} = \frac{1}{2}\int_{a_j}^{a_{j+1}} EI_j\left(\frac{\partial^2 \Psi^L{}_i}{\partial x^2}\right)^2 dx \tag{4}$$

$$F_{ij}{}^L = \frac{U^L{}_{ij}}{U^L{}_j} \text{ , } \sum_{j=1}^{N_d} F_{ij}{}^L = 1 \tag{5}$$

And

$$\sum_{j=1}^{N_d} F_{ij}{}^L = \sum_{j=1}^{N_d} F_{ij}^U = 1 \tag{6}$$

Where superscript, L indicates the loaded case

Selecting very small sub divisions, the flexural rigidity for j^{th} sub region, EI is equal for the both loaded and unloaded cases considered. Consequently, $F_{ij}{}^L$ can be written as

$$F_{ij}{}^L = \frac{(EI)_j \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \Psi^L{}_i}{\partial x^2}\right)^2 dx}{U^L{}_i} \tag{7}$$

The fractional energy varies in the element with sub regions. Differential fractional energy of the loaded and unloaded cases for a single location at the sub region j can therefore be written as follows to capture the energy change due to the axial load.

$$F_{ij}{}^L - F_{ij}{}^U = \frac{(EI)_j \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \Psi^L{}_i}{\partial x^2}\right)^2 dx}{U^L{}_i} - \frac{(EI)_j \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \Psi^U{}_i}{\partial x^2}\right)^2 dx}{U^U{}_i} \tag{8}$$

The above equation can be written by incorporating the all measured modes, m

$$F_{ij}^{Lm} - F_{ij}^{Um} = \sum_{i=1}^{m} \left(\frac{(EI)_j \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \Psi^L{}_i}{\partial x^2} \right)^2 dx}{U^L{}_i} \right) - \sum_{i=1}^{m} \left(\frac{(EI)_j \int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \Psi^U{}_i}{\partial x^2} \right)^2 dx}{U^U{}_i} \right) \quad (9)$$

The flexural rigidity, EI is a constant for the loaded and unloaded cases when the axial load applies in the linear elastic region. Equation 9 can hence be written as follows by introducing a novel vibration based parameter called Strain Energy Index, SEI. This parameter captures influence of the axial force using the modal vectors. It has been identified (will be presented later) from this research that SEI is inversely proportional to the axial deformation due to the axial force.

$$SEI = log \left(\sum_{i=1}^{m} \left(\frac{\int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \Psi^L{}_i}{\partial x^2} \right)^2 dx}{U^L{}_i} \right) - \sum_{i=1}^{m} \left(\frac{\int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \Psi^U{}_i}{\partial x^2} \right)^2 dx}{U^U{}_i} \right) \right) \quad (10)$$

The spline technique calculates partial derivatives consisting of the defined parameter, Stain Energy Index (SEI). Pervious researches have been used this technique to calculate the partial derivates and confirmed its accuracy (Choi et al, 2008).

SEI can be implemented for an element of a structure as described below.

01. The modal parameters such as natural frequencies and modal vectors can be obtained numerically(Finite Element Model (FEM)) for the structure without axial loads using the free vibration analysis

02. Ambient measurements can be extracted from accelerometers deployed on the structure.

03. Using these modal parameters, the FEM can be validated and F_{ij}^U (modal strain energy at unloaded case) for an element can be calculated using Equation 3 and stored for later use.

04. The above steps can be repeated in order to improve the model validation by applying known axial loads to both the FEM and the real structure,

05. The validated FEM of the structure is used to develop a data base/ graphs relating the parameter , SEI to the axial deformation (AD) as follows.

06. For a given axial load applied to the FEM of the structure, the modal parameters are determined using the modified program and the F_{ij}^L (modal strain energy at loaded case) is calculated using Equation 5. SEI for the particular case is calculated using Equation 10 using F_{ij}^U and F_{ij}^L determined previously. Axial deformation due to the axial force can also be obtained from static analysis. Repeating this procedure for a range of axial loads, a database for SEI vs AD can be developed.

07. Using the results from this database, graphs with the vertical axis representing the Strain Energy Index (SEI) and the horizontal axis representing the axial deformation (AD), can be plotted for each element in the structure.

This research identified that the variation of SEI with AD is linear. If SEI is known, the axial deformation AD can be obtained by applying either interpolation or extrapolation methods. During the service life of the structure, the axial deformation (AD) of any element can be obtained from the SEI vs AD graphs, if the SEI is known. Under an unknown axial load on the real structure, the modal parameters amend due to the stiffness matrix change, as addressed above, and these parameters can be extracted from the deployed accelerometers and then used to calculate SEI as described earlier.

ILLUSTRATIVE EXAMPLE

In this section, two numerical examples are used to present the capability of the proposed method of different structures.

Examples 1

A 0.5 x 0.5 x 4 m column was selected to study the proposed methodology. The material properties of the column are tabulated below (see Table 1). 10 axial compressive loading cases with loads such as 1, 2, 3 up to 10 MN are applied to the column to deform from the significant amount by maintaining the linear elastic region and first three vibration modes have been studied. The boundary conditions of the column have also been changed to examine their influence on the proposed parameter, Strain Energy Index, SEI. The columns with the different boundary conditions such as case A and case B are presented in Fig 1

Table 1: material properties of the column

Material Property	Numerical Value
Density/(kNm-3)	2300
Poisson Ratio	0.2
Young's Modulus /(GPa)	30

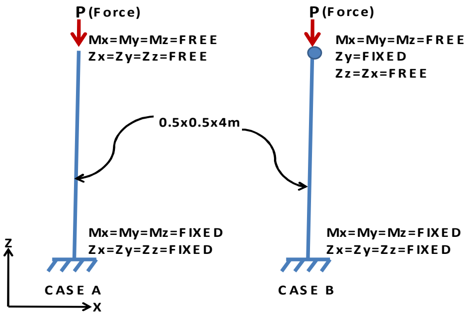
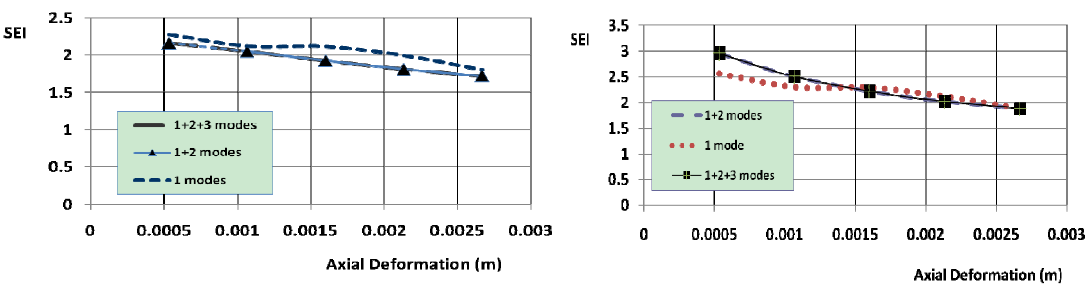


Fig 1: The columns with two different boundary conditions

Case A and Case B: Results and discussion

The first three vibration modes (two bending and one torsional) were used for the calculation. Additionally, percentages of the frequency change of the corresponding modes due to the axial force were also studied due to the fact that change of modal vectors can be examined using these percentages. It is interesting to note from the results that the percentages of the first two modes are higher than the others concluding that change of modal vectors is also more pronounced for first two modes. Consequently, this change can be used to capture influence of axial force through SEI.

After applying the axial force, modal vectors are calculated using data extracted from the analysis and Strain Energy Index; SEI is estimated using Equation 10. The corresponding axial deformation is estimated using static analysis. Having incorporated the first three vibration modes, SEI is calculated and results are presented in Fig 2. The impact of the number of modes on SEI is also studied while changing the number of modes incorporated into calculation of SEI. Fig 2 also shows variation of SEI with contribution of number of modes as well.



(a) (b)

Fig 2: Variation of SEI with the axial deformation –(a)-case A and (b)-case B

Fig 2 reveals that the gradients decrease with increasing the axial deformation confirming that SEI is inversely proportional to the axial deformation. SEI calculated incorporating only first mode does not show a smooth variation. Moreover, it is evident from this Figure that the impact of the number of modes on the SEI is not more pronounced when incorporated more than two modes with high natural frequencies. Consequently, SEI can be calculated incorporating first two modes to improve the accuracy of the proposed methodology. A slight variation of SEI for cases A and B can be observed highlighting impact of the boundary conditions on the modal vectors. .

Examples 2

As second example, two storey 2D structural framing system revealed in Fig 3 with the different axial loads of P1,P2,P3 and P4 applied on four columns such as L1, L2, R1 and R2 respectively is selected to study capabilities of SEI of these columns. The column sizes and the material properties of this system were the same as the previous example to provide equal stiffness of these columns before applying the axial forces. Consequently, the SEI variation due to the axial force can be purely investigated. The variation of the axial loads and the corresponding cases are tabulated in Table 2. These loading cases are defined to simulate behavior of columns in different levels of structural framing system subjected to different axial loads from different tributory areas. This simulation facilitates to study bahaviour of such columns of geometrically complex strucutral framing systems.

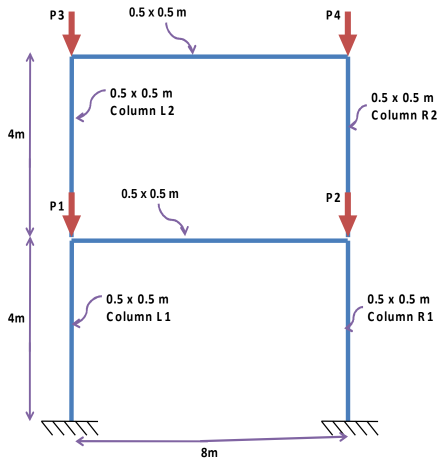
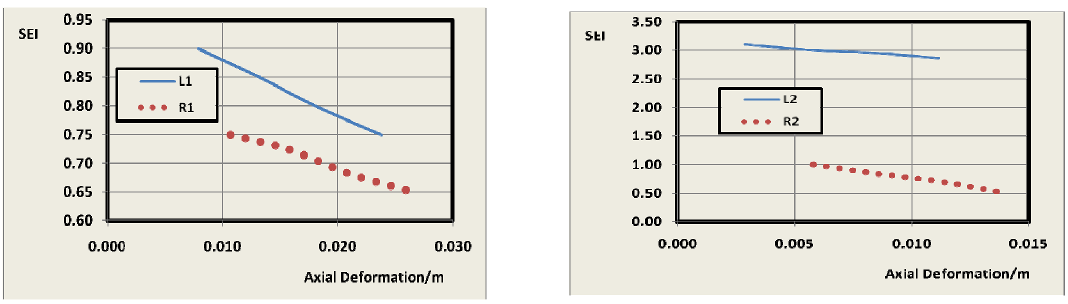


Table 2: The applied axial loads for the columns

	Case			
Loads(MN)	1	2	3	4
P1	10	15	20	25
P2	10	15	20	25
P3	5	10	15	20
P4	10	15	20	25

Fig 3: Two storey structural framing system

Modal vectors of the first two modes were selected to calculate SEI due to the fact that other modes with high natural frequencies do not give significant impact on SEI variation as observed in the previous example. Fig 4 (a) shows the variation of the SEI(s) of columns; L1 and R1 in the first storey while Fig 4 (b) shows variation of the SEI(s) of columns; L2 and R2 in the second storey. It is clear from Fig 4 that the SEI(s) of all the columns decrease lineally with increasing the axial deformation as experienced from in the previous example.



(a) (b)

Fig 4: Variation of SEI with the axial deformation of the selected structural elements

Fig 4(a) shows that there is a significant difference between the SEI(s) of columns L1 and R1 even though the same axial loads were applied on these two columns (see Table 2). This is because axial loads on columns in the second storey(L2 and R2) are different (see Table 2) so that these two different loadings impact on the bahaviour of columns in the first storey. This concludes that modal vectors and hence SEI can be used to capture such a complex bahaviour accurately. SEI of column L2 is higher than that of column R2 as observed in Fig 4(b) due to the fact that axial load applied on column L2 is smaller than column R2 (see Table 2). Furthermore, the difference between SEI(s) of columns L2 and R2 is higher than the other two columns in the first storey. This is because the percentage difference of axial force between columns L2 and R2 is higher in comparison to columns L1 and R1.

CONCLUSION

Many methods have been developed to quantify the axial loads and hence axial deformations of individual structural members using their natural frequencies, however, these methods can not be applied to individual members in structural framing systems as the natural frequency is a global parameter for the entire framing system. This highlights the need for a method to quantify axial deformations of elements of a structural framing system. A comprehensive method incorporating a vibration based parameter called Strain Energy Index (SEI) is proposed in this paper and illustrated through examples to examine capabilities of SEI. Results indicate that the proposed procedure with the parameter, SEI has an ability to quantify axial deformations of elements in structural framing system capturing the effects of the magnitudes of axial loads, the boundary conditions and the tributary areas supported by the elements.

REFERENCE

ANSYS 11.0 Manual, Ansys Inc., Canonsburg, PA

Banerjee, J.R,2000, 'Explicit modal analysis of an axially loaded Timoshenko beam with bending-torsion coupling', *Journal of Applied Science*, vol 67/307, pp 307-313

Choi,F.C, Li, J., Samali, B. , Crews,K., 2008, Application of the modified damage index method to timber beams, *Journal of Engineering Structures* , 30,pp 1124–1145

Della, C, N. & Shu, D. ,2009, 'Free vibration analysis of multiple delaminated beams under axial compressive loads', *Journal of Reinforced Plastics and Composites*, 28, pp 1365-1380

Stubbs, N.Kim, J.T, Topole, K.,1992, 'Ancient and robust algorithm for damage localization in offshore platforms', *Proceedings of the ASCE Tenth Structures Congress*, pp 543-546

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